A New Concept of Static Stability and Its Flight Testing in Supersonic Flight

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The current concept of static stability is not adequate for supersonic flight since it does not account for the influence of forces and moments due to altitude perturbations, which have a significant effect on the longitudinal motion. A new concept is proposed that accounts for altitude influence in an adequate manner. This concept, which is closely related to constant energy consideration, is based on a dynamic stability analysis with particular reference to the exponential characteristic modes of the airplane. Furthermore, it is shown that the well-known relation between static stability and the variation of elevator angle with speed is not valid in supersonic flight. As a consequence, the flight test methods currently used for determining static stability also are not adequate for supersonic flight. A new flight test method is proposed that provides an indication of static stability using the variation of elevator angle with altitude (or dynamic pressure, respectively). The points addressed may be of particular significance in regard to existing flying qualities requirements and criteria.

Nomenclature

$A(s)$	= coefficient matrix of the homogenous system
B	= scaling matrix for control inputs
B,C,D,E,F	= coefficients of the characteristic equation
C_D	= drag coefficient
C_{D_V}	$= \partial C_D / \partial (V/V_0)$
C_{D_α}	$= \partial C_D / \partial \alpha$
C_{D_δ}	$= \partial C_D / \partial \delta_e$
C_L	= lift coefficient
C_{L_V}	$= \partial C_L / \partial (V/V_0)$
C_{L_α}	$= \partial C_L / \partial \alpha$
C_{L_δ}	$= \partial C_L / \partial \delta_e$
C_m	= pitching moment coefficient
C_{m_q}	$= 2\partial C_m / \partial (q\bar{c}/V)$, pitch damping
C_{m_V}	$= \partial C_m / \partial (V/V_0)$, speed-dependent pitching moment
C_{m_α}	$= \partial C_m / \partial \alpha$, angle-of-attack stability
$C_{m_{\dot\alpha}}$	$= 2\partial C_m / \partial (\dot\alpha \bar{c}/V)$
C_{m_δ}	$= \partial C_m / \partial \delta_e$
C_{m_ρ}	= altitude-dependent pitching moment, $\mathrm{d}C_m = C_{m_\rho} \rho_h \mathrm{d}h$
$\bar{c}$	= mean aerodynamic chord
g	= acceleration due to gravity
h	= altitude
i_y	= radius of gyration
k_ρ	$= -g/(\rho_h V_0^2)$
M	= Mach number
n_V	= exponent of thrust speed dependence
n_ρ	= exponent of thrust altitude dependence
$\bar{q}$	$= (\rho/2)V^2$, dynamic pressure
s	= Laplace operator
t^*	$= \bar{c}/V_0$
V	= airspeed
x	= variable vector
α	= angle of attack
γ	= flight path angle
μ	$= 2m/(\rho S \bar{c})$
δ_e	= elevator angle
ρ	= air density
ρ_h	$= (1/\rho_0)\,\mathrm{d}\rho/\mathrm{d}h$, density gradient
Subscript	
0	= trim condition

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Introduction

STATIC stability, as is well known, represents a basic concept in flight mechanics and is one of the primary parameters for flying qualities (e.g., see Refs. 1-4). It provides a stability criterion based on the constant term in the characteristic equation valid for the case of constant air density or, as utilized in specifications such as MIL-F-8785B (Ref. 4), on the variation of elevator angle with speed. This concept was developed at a time when the airplanes were only capable of flying at subsonic speeds (e.g., see Ref. 5). Now that the flight regime has been extended to supersonic speeds, this concept is applied here, too. In contrast to subsonic flight, however, forces and moments caused by altitude perturbations have a significant influence on longitudinal stability in supersonic flight. In particular, they are much more significant than forces and moments caused by speed changes. This fact has been well recognized as far as dynamic stability is concerned (Refs. 1, 6-8). In regard to static stability, however, no such effects are taken into account since, as mentioned above, the concept currently used considers air density to be constant. It is the purpose of this paper to propose a static stability concept adequate for supersonic flight that accounts for altitude influence in an appropriate manner.

Another point of interest relates to the flight test methods for determining static stability. Two methods are currently used: the stabilized-airspeed method and the acceleration-deceleration method. It is not known which of them most accurately measures static stability in high-speed flight (Ref. 9). This is because, in high-speed flight, large altitude changes accompany small airspeed changes during tests using the stabilized-airspeed method, whereas altitude is held constant when applying the acceleration-deceleration method. It will be shown in this paper that neither method gives an indication of static stability in supersonic flight. Instead, a new method is proposed.

Current Concept

The current concept of static stability may best be described as it is defined in existing requirements for flying qualities such as MIL-F-8785B (Ref. 4). Here, static stability is related to the variation of elevator angle with speed in constant altitude $\mathrm{d}\delta_e/\mathrm{d}V|_{\rho=\mathrm{const}}$. The airplane is considered statically stable if $\mathrm{d}\delta_e/\mathrm{d}V|_{\rho=\mathrm{const}}$ is positive. Otherwise, it is considered unstable. The gradient $\mathrm{d}\delta_e/\mathrm{d}V|_{\rho=\mathrm{const}}$ is related to the constant term $E|_{\rho=\mathrm{const}}$ of the characteristic equation according to (e.g., Ref. 1)

$$\left.\frac{d\delta_e}{dV}\right|_{\rho=\text{const}} = -\frac{1}{V_0}\frac{2C_{L_0}/K_E}{C_{m_\delta}-C_{L_\delta}C_{m_\alpha}/C_{L_\alpha}}E|_{\rho=\text{const}} \quad (1)$$

where

$$K_E = 2g^2 C_{L_\alpha}/(\mu i_y^2) \quad (2)$$

The subscript $\rho=$const denotes the fact that the effects of altitude perturbations on stability are not accounted for, i.e. air density and speed of sound are considered constant. The coefficient $E|_{\rho=\text{const}}$ may be expressed as

$$E|_{\rho=\text{const}} = -K_E\left[\frac{C_{m_\alpha}}{C_{L_\alpha}}\left(1+\frac{C_{L_V}}{2C_{L_0}}\right)-\frac{C_{m_V}}{2C_{L_0}}\right] \quad (3)$$

As an equivalent criterion, the static margin $dC_m/dC_L|_{\rho=\text{const}}$, originally presented in extended form in Ref. 5, is often used (Refs. 1-3). It is related to the coefficient $E|_{\rho=\text{const}}$ according to

$$E|_{\rho=\text{const}} = -K_E\left.\frac{dC_m}{dC_L}\right|_{\rho=\text{const}} \quad (4)$$

The relations described are applied to subsonic as well as to supersonic flight. They are generally considered adequate for supersonic flight. It is stated in Ref. 1 only that – generally speaking for a linear/invariant system of arbitrary degree – the constant term in the characteristic equation provides a criterion for static stability. However, no specific investigation exists as to what this means in supersonic flight.

The points addressed may be of particular interest in regard to flight standards especially developed for supersonic vehicles, such as Refs. 10 and 11. Here, stick-free static stability is specified which, in case of irreversible control systems, is equivalent to stick-fixed stability outlined above.

Dynamic Stability Analysis and Constant-Term Concept

It is well known that altitude perturbations have a significant influence on dynamic stability of horizontal flight when flying at supersonic speed (Refs. 1, 3, 6-8). In linearized form, the equations of motion for longitudinal flight may be written as

$$A(s)x(s) = -B\delta_e(s) \quad (5a)$$

where

$$A(s) = \begin{bmatrix} s\mu t^* + (2+C_{D_V}/C_{D_0}-n_V)C_{D_0} & C_{D_\alpha} & st^*C_{L_0}+\bar{c}\rho_h C_{D_0}(1-n_\rho) \\ 2C_{L_0}+C_{L_V} & C_{L_\alpha}+C_{D_0} & -\mu(st^*)^2+\bar{c}\rho_h C_{L_0} \\ C_{m_V} & -\mu(st^*)^2(i_y/\bar{c})^2 + st^*(C_{m_q}+C_{m_{\dot\alpha}})/2+C_{m_\alpha} & -\mu(st^*)^3(i_y/\bar{c})^2 + (st^*)^2C_{m_q}/2+\bar{c}\rho_h C_{m_\rho} \end{bmatrix} \quad (5b)$$

$$x^T(s) = [\Delta V/V_0 \;\; \Delta\alpha \;\; \Delta h/\bar{c}] \quad (5c)$$

$$B = \begin{bmatrix} C_{D_\delta} \\ C_{L_\delta} \\ C_{m_\delta} \end{bmatrix} \quad (5d)$$

Due to the effect of altitude perturbations, the characteristic equation is a quintic rather than a quartic:

$$s^5+Bs^4+Cs^3+Ds^2+Es+F=0 \quad (6)$$

The constant term F may be written as

$$F = K_F(a_1C_{m_\alpha}/C_{L_\alpha}+a_2C_{m_\rho}/C_{L_0}+a_3C_{m_V}/C_{L_0}) \quad (7)$$

where K_F represents a positive constant. The coefficients $a_{1,2,3}$ are determined by thrust and drag characteristics. Applying Ackeret's rule [i.e., $C_{D_V}/C_{D_0}=C_{L_V}/C_{L_0}=-M_0^2/(M_0^2-1)$], the coefficients may be written as

$$a_1 = \{n_V - n_\rho[2-M_0^2/(M_0^2-1)]\}C_{D_0}/C_{L_0} \quad (8a)$$

$$a_2 = -n_V C_{D_0}/C_{L_0} + [2-M_0^2/(M_0^2-1)] \times (C_{D_0}/C_{L_0}-C_{D_\alpha}/C_{L_\alpha}) \quad (8b)$$

$$a_3 = -(1-n_\rho)C_{D_0}/C_{L_0}+C_{D_\alpha}/C_{L_\alpha} \quad (8c)$$

The coefficients n_V and n_ρ denote the variation of thrust T with speed (Mach number) and altitude (density, temperature) according to

$$T = T_0(V/V_0)^{n_V}(\rho/\rho_0)^{n_\rho} \quad (9)$$

A static stability concept based on the constant term F is not adequate for supersonic flight for various reasons. One of the main reasons refers to the relation between angle-of-attack stability C_{m_α} and the nature of the exponential characteristic modes. A dynamic stability analysis (Ref. 12) shows that positive values of C_{m_α} *always* lead to an aperiodic divergence, as illustrated in Fig. 1.† The constant term F may be positive or negative in this case. This depends on thrust and drag characteristics, as expressed by the quantity a_1 in Eqs. (7) and (8a). Increase of thrust with speed ($n_V>0$), as is the case for supersonic vehicles, leads to positive values of F when C_{m_α} is unstable ($C_{m_\alpha}>0$). This means that the constant term F would indicate positive static stability despite an aperiodic divergence seriously impairing the handling qualities.

On the other hand, F may be negative when C_{m_α} is stable ($C_{m_\alpha}<0$). In this case, an aperiodic divergence exists that is characterized by a very small root. This root denotes a characteristic mode known as "height mode," which is significant only in supersonic flight. It exists in addition to the phygoid and the short-period mode (e.g., see Refs. 1, 6-8). The order of magnitude of the height mode root is $(g/V_0)\times C_{D_0}/C_{L_0}$, which is 10^{-3} sec^{-1} for a flight at a Mach number of $M_0=3$. This is valid for arbitrary values of C_{m_α} (see Fig. 1). Therefore, the instability represented by the height mode root (i.e., by the constant term F) can be disregarded when compared with an aperiodic divergence caused by an unstable C_{m_α} value. As a result, the constant term F is of no practical use as an indicator of aperiodic divergences, and it cannot provide the stability criterion classically assigned to the concept of static stability from a flying qualities point of view. This is of particular significance for existing requirements of flying qualities such as MIL-F-8785B where, as stated in its supporting document (Ref. 9), the primary purpose of the static stability paragraph is to prevent aperiodic divergences.

†Data used here and in the following figures pertain to the Mach 3 supersonic transport described in Ref. 6.

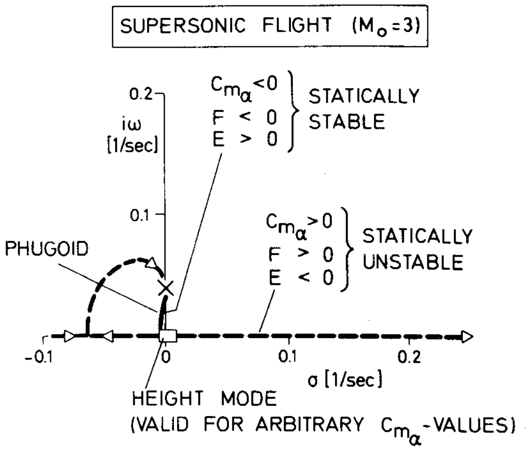

Fig. 1 Effect of C_{m_α} on dynamic stability (phugoid and height mode).

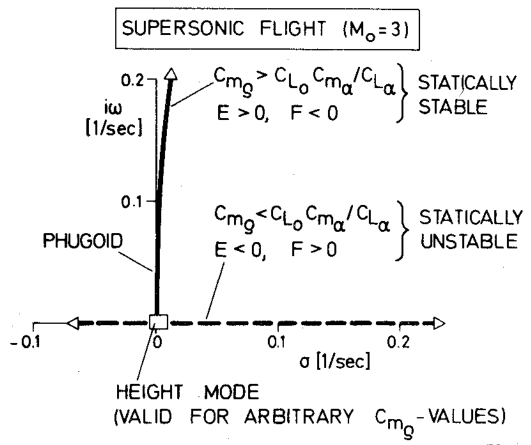

Fig. 2 Effect of C_{m_ρ} on dynamic stability (phugoid and height mode).

A criterion for the correlation between unstable values of C_{m_α} and aperiodic divergences can be based on the coefficient E of the stability quintic. This is due to the fact that E is entirely determined by C_{m_α} (and by the other static moments C_{m_ρ} and C_{m_V}). The coefficient E, when neglecting all insignificant parts, may be written as

$$E = -K_E^* \left(\frac{C_{m_\alpha}}{C_{L_\alpha}} - \frac{C_{m_\rho} + k_\rho C_{m_V}}{C_{L_0}} \right) \tag{10}$$

where K_E^* is a positive constant (i.e., $K_E^* = K_E/(2k_\rho)$, with K_E given by Eq. (2) and $k_\rho = -g/(\rho_h V_0^2)$). This means for the case considered above ($C_{m_\rho} = 0$, $C_{m_V} = 0$), that E changes its sign when C_{m_α} introduces an aperiodic divergence (see also Fig. 1). As a result, the coefficient E can be used as an indicator of static stability concerning the effect of C_{m_α}.

The two other moments affecting static stability are C_{m_ρ} and C_{m_V}. They denote pitching moment changes caused by altitude and speed perturbations according to

$$\Delta C_m(\Delta h) = C_{m_\rho} \Delta\rho/\rho_0 = C_{m_\rho} \rho_h \Delta h \tag{11a}$$

$$\Delta C_m(\Delta V) = C_{m_V} \Delta V/V_0 \tag{11b}$$

A dynamic stability analysis (Ref. 12) shows that the nature of the exponential characteristic modes associated with C_{m_ρ} and C_{m_V} can be described adequately by the behavior of the coefficient E. This means that there is *always* an aperiodic divergence when the coefficient E changes its sign due to values of C_{m_ρ} and C_{m_V} more negative than $C_{L_0} C_{m_\alpha}/C_{L_\alpha}$,

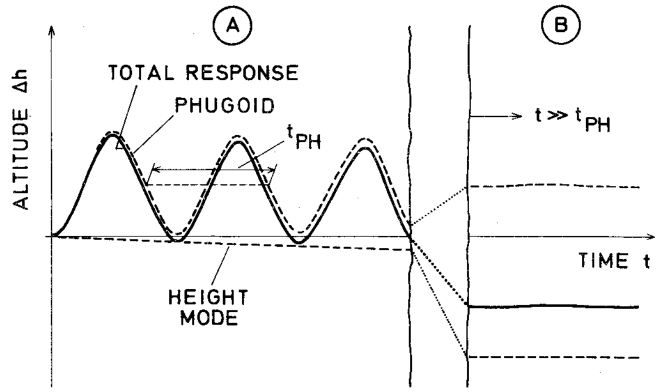

Fig. 3 Altitude response to step elevator input in supersonic flight.

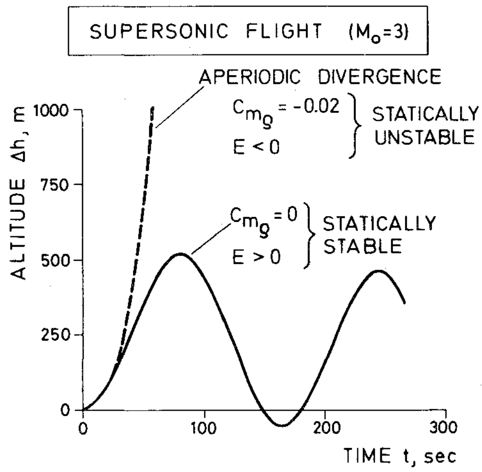

Fig. 4 Effect of C_{m_ρ} on altitude response to step elevator input.

i.e., due to

$$C_{m_\rho} + k_\rho C_{m_V} < C_{L_0} C_{m_\alpha}/C_{L_\alpha} \tag{12}$$

This is illustrated in Fig. 2 using the effect of C_{m_ρ} as an example. As a result, the coefficient E can also be used as an indicator of static stability in the more general case where C_{m_ρ} and C_{m_V} exist. Here again, a criterion based on the sign of the constant term F fails to indicate the aperiodic divergence caused by C_{m_ρ} and C_{m_V}, because, depending on thrust and drag characteristics as described by Eqs. (8b, c), F may be positive when unstable values of C_{m_ρ} and C_{m_V} lead to an aperiodic divergence (Fig. 2).

Constant-Energy Concept and Static Margin

The criterion based on the coefficient E of the stability quintic can be regarded as an extension of the classical concept described by Eqs. (3) and (4) since it includes the effects of altitude perturbations on stability. As will be shown in the following, altitude perturbations are included on the basis of a motion with a constant sum of kinetic plus potential energy. This can be utilized to introduce a generalized expression of the static margin dC_m/dC_L, which, by analogy to the current theory, relates the variation of pitching moment to lift coefficient.

A physical insight into the problem is given in Fig. 3, which shows the altitude response of the airplane to a step elevator input. The total response consists of a superposition of the phugoid and the height mode mentioned before. Since the height mode is significant in supersonic flight, the total time region may be separated into two parts A and B. Part B represents a time region in which altitude response approaches a value which can be considered strictly static. It is determined by the height mode. Its stable or unstable character is closely related to the constant term F of the characteristic equation, the properties of which have been discussed previously. Part A, on the other hand, represents an intermediate time region in which the phugoid is the predominant mode of motion. Here, altitude response is determined by the static moments C_{m_α}, C_{m_ρ}, and C_{m_V}. Because of this property and its intermediate nature, altitude response in this time region may be qualified as quasistatic. The stability characteristics, here, are determined by the coefficient E, which is closely related to the phugoid (Ref. 12). In particular, it is this time region where an aperiodic divergence caused by unstable values of the static moments becomes significant. This is illustrated in Fig. 4, using the effect of C_{m_ρ} and its correlation with the sign of E as an example. Due to the dominance of the phugoid, altitude response in this time region is characterized by the sum of kinetic plus potential energy being constant, i.e., by

$$V^2/2+gh=\text{const} \tag{13}$$

On the basis of this equation, it is now possible to relate altitude and speed perturbations to derive an expression of the static margin adequate for supersonic flight.

The change in pitching moment and lift coefficient for perturbations from a reference state may be expressed as

$$\mathrm{d}C_m = C_{m_\alpha}\mathrm{d}\alpha + C_{m_V}\mathrm{d}V/V_0 + C_{m_\rho}\rho_h \mathrm{d}h \tag{14a}$$

$$\mathrm{d}C_L = C_{L_\alpha}\mathrm{d}\alpha + C_{L_V}\mathrm{d}V/V_0 \tag{14b}$$

Two conditions must be imposed. The first one, by analogy with the current concept of static stability, presupposes that the perturbations $\mathrm{d}\alpha$, $\mathrm{d}V$ and $\mathrm{d}h$ develop in such a way that lift equals weight ($L=\mathrm{mg}$), with elevator angle and throttle setting constant. This yields $\mathrm{d}L=0$ or

$$C_{L_\alpha}\mathrm{d}\alpha + (2C_{L_0}+C_{L_V})\mathrm{d}V/V_0 + C_{L_0}\rho_h\mathrm{d}h = 0 \tag{15}$$

The second condition, as a new and an additional condition, refers to the relation between speed and altitude perturbations according to a constant energy motion. From Eq. (13) it follows that

$$\mathrm{d}V = -(g/V_0)\mathrm{d}h \tag{16}$$

Combining Eqs. (14), (15), and (16) and accounting for $k_\rho \ll 1$ (this, as will be shown later, is valid throughout the whole supersonic flight regime) yield the following expression for the static margin

$$\frac{\mathrm{d}C_m}{\mathrm{d}C_L} \approx \frac{C_{m_\alpha}}{C_{L_\alpha}} - \frac{C_{m_\rho}+k_\rho C_{m_V}}{C_{L_0}} \tag{17}$$

From Eq. (10) it follows that

$$E = -K_E^* \mathrm{d}C_m/\mathrm{d}C_L \tag{18}$$

Thus, the static margin defined by Eq. (17) provides the same stability criterion as the coefficient E of the stability quintic.

There are three significant differences when compared with the current concept of static stability expressed by Eqs. (3) and (4):

1) Angle-of-attack stability C_{m_α}: Change of lift with Mach number ($C_{L_V}=M_0\partial C_L/\partial M$) does not affect the influence of C_{m_α} on static stability. Applying Ackeret's rule ($\partial C_L/\partial M = -C_{L_0}M_0/(M_0^2-1)$), a difference in static margin of more than 100% exists when comparing $\mathrm{d}C_m/\mathrm{d}C_L|_{\rho=\text{const}}$ with the expression of Eq. (17).

2) Altitude-dependent pitching moments C_{m_ρ}, which for example may be caused by elastic deformations due to dynamic pressure perturbations, significantly affect static stability. The current concept completely ignores such effects.

3) Due to $k_\rho \ll 1$ in supersonic flight, speed-dependent pitching moments C_{m_V} are of negligible influence.

In the relations derived above, the quantity $k_\rho = -g/(\rho_h V_0^2)$ has been introduced, with $\rho_h = (1/\rho_0)\mathrm{d}\rho/\mathrm{d}h$. It has been assumed that $k_\rho \ll 1$ in supersonic flight. This is confirmed in Fig. 5, which shows that the relation $k_\rho \ll 1$ is valid for Mach-numbers greater than 2 at any altitude. It may be added that k_ρ represents the ratio of speed-dependent to altitude-dependent lift changes for a mode of motion with constant energy.

The static stability concept proposed in this paper is valid not only in supersonic flight but also in subsonic and low-speed flight. This can best be demonstrated for low-speed flight where k_ρ is very large (i.e., $k_\rho \gg 1$). In this case, the stability quintic of Eq. (5) including the effects of altitude perturbations can also be applied. The coefficient E or the static margin resulting from Eqs. (14)-(16) can be expressed for low-speed flight in the following form

$$\frac{\mathrm{d}C_m}{\mathrm{d}C_L} \approx \underbrace{\left(1+\frac{C_{L_V}}{2C_{L_0}}\right)\frac{C_{m_\alpha}}{C_{L_\alpha}} - \frac{C_{m_V}}{2C_{L_0}}}_{\mathrm{d}C_m/\mathrm{d}C_L|_{\rho=\text{const}}} - \frac{1}{2k_\rho}\frac{C_{m_\rho}}{C_{L_0}} \tag{19}$$

The first part on the right-hand side represents the static stability criterion currently used [according to Eqs. (3) and (4)]. The second part, which is combined with the factor $1/(2k_\rho)$, accounts for the influence of altitude perturbations as it exists in low-speed flight. Due to $k_\rho \gg 1$, this is always negligible here. As a result, the static stability concept proposed in this paper includes the current concept as a special case that is restricted to subsonic flight.

Elevator Angle Gradient

From the discussion in the previous sections, it follows that the variation $\mathrm{d}\delta_e/\mathrm{d}V|_{\rho=\text{const}}$ of elevator angle with speed as currently used cannot provide an indication of static stability in supersonic flight since it does not account for the effect of altitude perturbations on stability. Moreover, the variation of δ_e with V cannot be used, even if the constant-altitude constraint is removed. This is discussed in the following. In supersonic flight, two kinds of elevator angle gradient can be considered. The first is related to the variation of δ_e and V which is strictly static. From Eqs. (5) and (6) it follows that this can be expressed in the following form:

$$\frac{\mathrm{d}\delta_e}{\mathrm{d}V} = -\frac{C_{L_0}}{a_3 K_F V_0 C_{m_\delta}} F \tag{20}$$

with F and a_3 given by Eqs. (7) and (8c) and, for convenience, with C_{D_δ} and C_{L_δ} ignored. This means that the variation

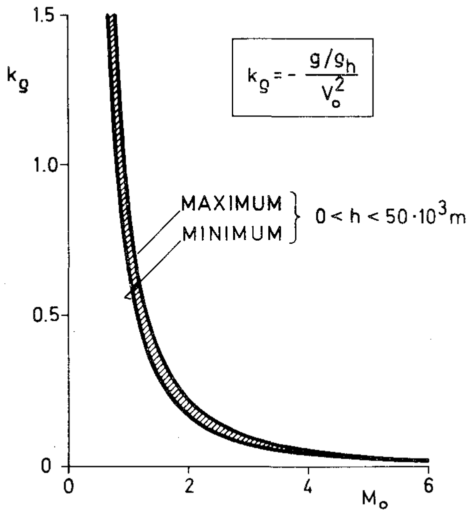

Fig. 5 Mach number vs k_ρ (atmospheric data from Ref. 13).

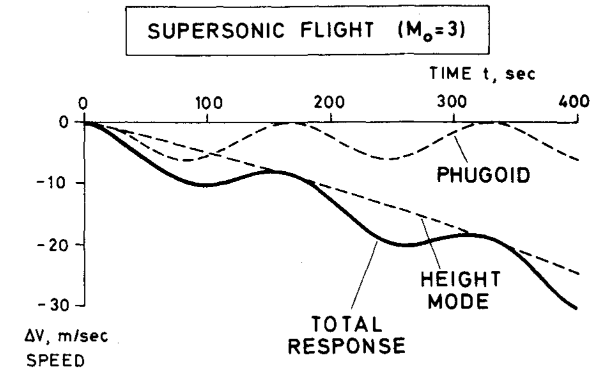

Fig. 6 Speed response to step elevator input.

strictly static is proportional to the constant term F, which, as has been shown before, does not provide an indication of static stability. The second kind of elevator angle gradient refers to the variation of δ_e and V on a quasistatic basis, as it exists for altitude response in the time region denoted by part A in Fig. 3. The quasistatic behavior has been qualified as the property that, for an intermediate time region, the response to an elevator input is determined by phugoid dynamics, i.e., by the static moments. In Fig. 6, speed response to a step elevator input is shown. It is evident that speed response is dominated by the height-mode contribution, with minor influence of the phugoid. This means that speed does not behave in a quasistatic manner. Thus, a gradient $d\delta_e/dV$ on a quasistatic basis would be without any meaning. As a result, the variation of elevator angle with speed does not provide an indication of static stability when flying at supersonic speed.

An explanation of the physical background may be given as follows. In supersonic flight, the phugoid (i.e., its eigenvector) can be considered as mainly an altitude oscillation, with only minor speed components. The ratio of speed to altitude variations is approximately equal to k_ρ, which, as shown in Fig. 5, is very small ($k_\rho \ll 1$). By contrast, the height mode is a mode of motion in which speed and altitude changes are of equal magnitude. This is due to the fact that the height mode can be considered as a motion with constant total lift, where the lift decrease due to speed decrease is compensated for by the lift increase due to altitude variations and vice versa. An elevator deflection excites both phugoid and height mode. In regard to speed response, however, the phugoid does not provide a significant contribution because it itself shows only minor speed changes. Therefore, the speed response is governed by the height mode contribution, despite the fact that this mode develops much more slowly than the phugoid.

Flight Test Methods

There are two methods currently used to determine static stability and to demonstrate compliance with related flying qualities requirements (Ref. 9). The first one, which may be called the stabilized-airspeed method, is to first trim the airplane and then use elevator control alone to change and restabilize airspeed. Altitude is not constant since it varies according to the disturbances generated by elevator deflections. This is particularly significant in high-speed flight where large altitude changes accompany small airspeed changes. The elevator angle versus airspeed gradient obtained from the stabilized-airspeed method provides an indication of static stability in subsonic flight, especially in the low-speed region. In supersonic flight, however, this is no longer valid. Here, the δ_e vs V gradient represents the change strictly static which, as shown before, is of no use.

The second method is the acceleration-deceleration technique. After trimming, the airplane is accelerated and decelerated within specified limits of the speed range by changing power. Altitude is held constant with the elevator. It is clear that this technique cannot account for the influence of altitude perturbations. In fact, the method provides a relation between elevator position and speed, which, when neglecting acceleration and deceleration effects and when properly accounting for off-trim power settings, yields the elevator gradient according to the current concept of static stability, i.e., according to Eq. (1). This means that its applicability is limited to the same extent as the current static stability concept itself and, thus, it is not adequate for supersonic flight.

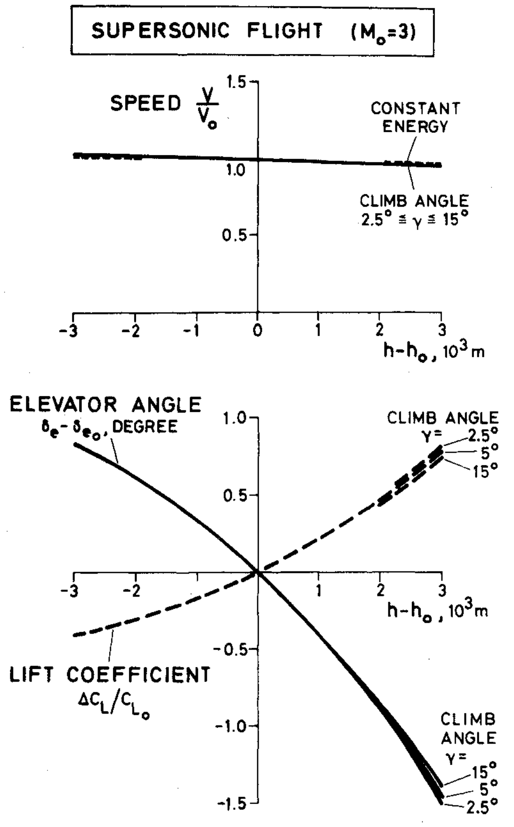

Fig. 7 Climb method: change of speed, elevator angle, and lift coefficient with altitude.

A flight test technique adequate for supersonic flight must have the following properties. First, altitude effects must be taken into account, with altitude related to speed on the basis of constant energy (quasistatic changes). Second, the changes of speed, altitude, and angle of attack must be such that lift equals weight, with load factor held constant in order to avoid unwanted contributions to lift and pitching moment. This can be accomplished by a climb with constant climb angle γ. Trimmer and throttle controls are held constant at their trim settings. The only control device necessary for performing the constant-climb-angle flight is the elevator, the variation of which gives an indication of static stability. The elevator is varied so as to increase angle of attack or lift coefficient to compensate for the lift loss due to dynamic pressure decrease caused by altitude increase and speed reduction. A numerical example including nonlinearities of force and moment characteristics is shown in Fig. 7. The constant-energy property of the climb is seen to exist. This is true for a very large altitude interval (5000 m) yielding an air density and dynamic pressure change of more than 100%. It is based on the high level of kinetic energy in supersonic flight due to which small changes of airspeed are equivalent to large changes of altitude. In Fig. 7, it is also shown how lift coefficient and elevator angle vary. The variation of elevator angle may be determined in the following way. The pitching moment equilibrium is given by

$$dC_m = 0 = C_{m_\alpha} d\alpha + C_{m_V} dV/V_0 + C_{m_\rho} \rho_h dh + C_{m_{\delta}} d\delta_e \quad (21)$$

The lift equation $dL=0$, accounting for climb angle γ, can be written as

$$C_{L_\alpha}d\alpha+(2C_{L_0}\cos\gamma+C_{L_V})dV/V_0 + C_{L_0}\cos\gamma\ \rho_h dh + C_{L_\delta}d\delta_e = 0 \qquad (22)$$

The drag equation $m\dot{V}=T-D-mg\sin\gamma$ may be expressed in the following form

$$dV=-\left(1-\frac{T-D}{mg\sin\gamma}\right)\frac{g}{V}dh$$

With the use of

$$D=(C_{D\min}+kC_L^2)(\rho/2)V^2S$$

the drag equation may be rewritten as

$$dV=-(1-kC_{L_0}\sin\gamma)(g/V_0)dh$$

Due to

$$kC_{L_0}\sin\gamma \ll 1$$

it follows that

$$dV=-(g/V_0)dh \qquad (23)$$

Combining this with Eqs. (21) and (22) and accounting for $k_\rho \ll 1$ yield the following expression

$$\frac{d\delta_e}{dh}=\rho_h\frac{(1+2k_\rho)C_{L_0}}{C_{m_\delta}-C_{L_\delta}C_{m_\alpha}/C_{L_\alpha}}\left[\cos\gamma\frac{C_{m_\alpha}}{C_{L_\alpha}}-\frac{C_{m_\rho}+k_\rho C_{m_V}}{C_{L_0}}\right] \qquad (24)$$

The term in brackets is equal to the static margin described by Eq. (17) when the following condition is imposed

$$\cos\gamma\approx 1$$

Thus

$$\frac{d\delta_e}{dh}\approx\rho_h\frac{(1+2k_\rho)C_{L_0}}{C_{m_\delta}-C_{L_\delta}C_{m_\alpha}/C_{L_\alpha}}\frac{dC_m}{dC_L} \qquad (25)$$

From this result, it follows that the constant-climb-angle method proposed yields a relation between δ_e and h, which provides an indication of static stability. As to Eq. (25), it has been considered appropriate to relate elevator angle to altitude change and not to speed. This is based on the fact that lift and moment changes during the climb are mainly due to density change. Speed change is very small (see Fig. 7). As a consequence, deviations from a constant-energy climb have only little influence when δ_e is related to h. But they introduce large errors when δ_e is related to V.

As an alternative, dynamic pressure $\bar{q}=(\rho/2)V^2$ may be used that is equivalent to altitude. This avoids the determination of density gradient $\rho_h=(1/\rho_0)d\rho/dh$ in Eq. (25). It yields the following expression (where, for convenience, C_{L_δ} has been dropped)

$$\frac{d\delta_e}{d(\bar{q}/\bar{q}_0)}\approx\frac{C_{L_0}}{C_{m_\delta}}\frac{dC_m}{dC_L} \qquad (26)$$

An example is shown in Fig. 8.

The condition $\cos\gamma\approx 1$ imposes an upper limit on the climb angle usable. Otherwise, the contribution of angle-of-attack stability to the stability margin dC_m/dC_L would be underrated, as indicated in Eq. (24). The effect of climb angle is included in the example of Figs. 7 and 8. From this it follows that there are practically the same results up to a climb angle of $\gamma=15°$. This means that there is a wide range of climb angles free to be chosen. There is also a lower climb angle limit. This is related to energy deviation. The deviation from constant energy depends on the difference of thrust minus drag during the climb and on flight pathlength. The flight pathlength necessary for a given altitude interval increases when climb angle γ is decreased. It approaches infinity when γ becomes zero. Due to the great flight pathlength resulting from small climb angles, energy can no longer be considered constant but may vary appreciably. This imposes a lower limit on the climb angle usable.

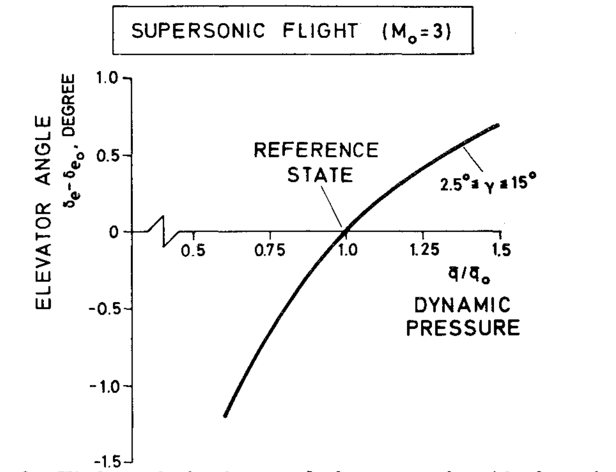

Fig. 8 Climb method: change of elevator angle with dynamic pressure.

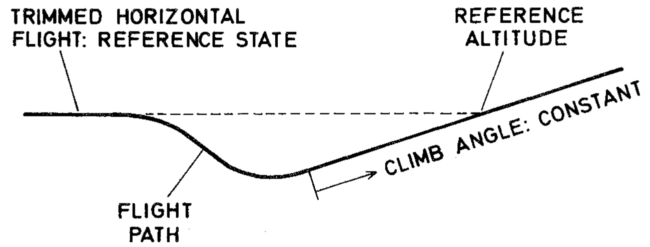

Fig. 9 Flight procedure (not to scale).

In the numerical examples of Figs. 7 and 8, the climb interval shown begins below the reference altitude. Initial conditions are chosen so as to meet the trim values when the airplane reaches the reference altitude. From a practical point of view, this may be accomplished by the procedure illustrated in Fig. 9. The airplane is trimmed in horizontal flight at the reference altitude. With throttle and trimmer controls held constant at their trim settings, the airplane descends to an altitude where the climb interval from which data should be taken begins. Here, the airplane starts the constant-climb-angle flight.

In regard to the constant-climb-angle technique described above, it is necessary for the pilot to get an indication of climb angle γ. This requires the measurement of γ by the difference of attitude angle and angle of attack or through the use of an inertial system. An alternative technique, which avoids the measurement of climb angle, is to hold climb rate constant. This is illustrated in Fig. 10, which shows that the constant-climb-rate technique yields the same results as the constant-climb-angle technique. Here again, it is possible to use a wide range of climb rates.

There are two points that may be of interest. The first is the effect of thrust on static stability. In order to account for this in a proper manner, it is necessary to hold throttle constant at its trim setting. This in no way presents any difficulties for the proposed climb method, since the only control device necessary and sufficient for performing the constant-climb-angle or constant-climb-rate flight is the elevator. As to the

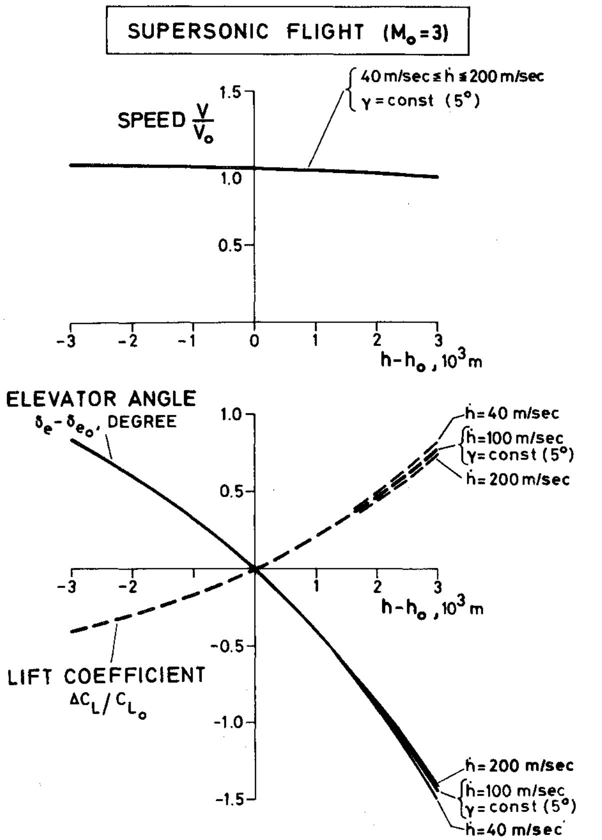

Fig. 10 Climb method: comparison of constant-climb-rate and constant-climb-angle flight.

second point, the method proposed for flight testing for static stability in supersonic flight requires the airplane to climb. The purpose is to vary altitude to account for its effect on static stability. This can also be achieved by a descending flight path. Here, the pilot varies altitude descending either with a constant negative flight-path angle or with a constant descent rate. Everything else is analogous to the climb technique.

Conclusion

A new concept of static stability is presented that is adequate for supersonic flight. Based on a dynamic stability analysis with particular reference to the exponential characteristic modes, it accounts for the substantial influence exerted by altitude perturbations on the longitudinal motion in high-speed flight. It is closely related to flight conditions where the sum of kinetic plus potential energy is constant. This concept is valid not only in supersonic flight but also in subsonic and low-speed flight. Furthermore, it is shown that the variation of elevator angle with speed does not provide an indication of static stability when flying at supersonic speeds. Therefore, flight test methods currently known for determining static stability are not adequate for supersonic flight. A new flight test method is proposed based on a constant climb-angle or a constant-climb-rate flight. It utilizes the fact that, in supersonic flight, the variation of elevator angle with altitude provides an indication of static stability. The points addressed may be of particular significance in regard to flying qualities criteria or requirements as far as static stability is concerned.

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